RESEARCH STATEMENT

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1. Overview

I am interested in algebraic geometry and related areas. My current research work is centered on the geometry of $K3$ surfaces and cubic fourfolds, especially their moduli theory and automorphism groups.

A complex $K3$ surface is a simply connected smooth compact complex surface with an everywhere non-degenerate holomorphic two form. The global Torelli theorem holds for $K3$ surfaces ([PŠ71],[BR75],[LP81]). An insightful construction by Beauville-Donagi [BD85] showed that the Fano scheme of lines on a smooth cubic fourfold is a hyper-Kähler fourfold, which is deformation equivalent to the Hilbert scheme of two points on a $K3$ surface. Later on, Voisin proved the global Torelli theorem for cubic fourfolds ([Voi86]). More relations between cubic fourfolds and $K3$ surfaces were investigated from both perspectives of Hodge theory ([Has00]) and derived category ([Kuz10]).

An important problem is to classify the automorphism groups of $K3$ surfaces and cubic fourfolds. A celebrated result from Mukai ([Muk88]) is that there are 11 maximal finite groups of symplectic automorphisms of K3 surfaces. More precisely, these 11 groups are exactly those subgroups of the Mathieu group M_{23} with 5 orbits in their induced action on $\{1, 2, ..., 24\}$. However, the classification of automorphism groups of cubic fourfolds is far from established. An automorphism of a cubic fourfold X is called symplectic if it acts trivially on $H^{3,1}(X)$. In my work with Radu Laza [LZ18], we classified all symplectic automorphism groups of smooth cubic fourfolds.

Theorem 1.1 (Laza-Zheng). *There are 34 finite groups which can be realized as symplectic automorphism groups of smooth cubic fourfolds, detailed description can be find in theorem 3.2.*

Moduli theory is a central theme in algebraic geometry. The key notion in moduli theory is moduli space, which describes a universal family of geometric objects of a given type, in the sense that any other such family is realized inside it. So any structure such a moduli space might possess, is inherited by all other families of such objects. Conversely, anything that all these objects (or families of such) have in common is passed on to the moduli space.

There are three standard approaches to construct and compactify moduli spaces in algebraic geometry. The most accessible approach is geometric invariant theory. The second approach is using period map in Hodge theory. The third approach is the construction of KSBA compactification via minimal model program. These three approaches are related to each other, and form a very active and fascinating research area in modern algebraic geometry.

For polarized $K3$ surfaces, cubic fourfolds or certain objects related to them, the Hodge theoretic approach behaves very well thanks to the global Torelli theorems. One can either look at GIT

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constructions, or realize moduli spaces as locally symmetric varieties via the period maps. It is then natural to study the GIT-compactifications and Baily-Borel compactifications, and ask for relations between them. In many cases, the images of the moduli of smooth geometric objects under the period maps are complements of hyperplane arrangements in arithmetic quotients of Hermitian symmetric domains (either balls or type IV domains). For examples, the moduli spaces of smooth cubic surfaces ([ACT02]), smooth cubic threefolds ([ACT11], [LS07]), smooth quartic curves ([Kon00]) and non-hyperelliptic curves of genus 4 ([Kon02]) are realized as arrangement complements in balls of dimensions $4, 10, 6, 9$ respectively. These four cases were considered again in [KR12] via families of abelian varieties with extra structures.

Inspired by the work of Shah ([Sha80]), Looijenga ([Loo03a, Loo03b]) constructed a compactification for an arrangement complement in an arithmetic quotient of ball or type IV domain. This is now called Looijenga compactification, and it coincides with Baily-Borel compactification if the hyperplane arrangement is trivial. In many situations (including cases of cubic fourfold [Loo09],[Laz10], cubic surface [ACT02], cubic threefold [ACT11],[LS07], quartic curve [Kon00], genus 4 curve [Kon02], rational elliptic surface [HL02] and so on), the GIT-compactifications are identified with the Looijenga compactifications via extensions of the period maps.

In my work with Chenglong Yu [YZ18b], we showed:

Theorem 1.2 (Yu-Zheng)**.** *The moduli space of cubic fourfolds with a specified action of a finite group is isomorphic to an arithmetic quotient of a Hermitian symmetric domain (either a ball or a type* IV *domain). Moreover, the* GIT*-compactification and Looijenga compactification are naturally identified, and a criterion about the finite group action is given on when the Looijenga compactification is actually Baily-Borel compactification. See theorem 4.1 for more explicit statement.*

In [YZ18a], we extended the methods in [YZ18b] to obtain new results for singular sextic curves, see theorem 4.2.

2. Occult period maps

2.1. **Conjectures by Kudla and Rapoport.** My first research topic was on a series of conjectures made by Kudla and Rapoport in [KR12] (remark 5.2, 6.2, 7.2, 8.2). In their paper, Kudla and Rapoport interpreted what they called occult period maps as morphisms between moduli stacks arising as GIT quotients and moduli stacks of abelian varieties with additional structures. Explicitly, the occult period map sends a smooth cubic threefold to the polarized Hodge structure of a cubic fourfold which is defined as the triple cover of $\mathbb{P}^4$ branched along the cubic threefold. This realizes the moduli of smooth cubic threefolds as an arrangement complement in an arithmetic ball quotient of dimension 10. There are also occult period maps for cubic surfaces, quartic curves and non-hyperelliptic curves of genus 4, which realized the moduli spaces as arrangement complements in arithmetic ball quotients of dimension $4, 6, 9$ respectively. Let $\mathcal{M}$ be the moduli space (constructed by GIT) in one of these four cases, and $\Gamma\backslash\mathbb{B}$ the corresponding ball quotient. Denote $\mathscr{P}\colon \mathcal{M} \longrightarrow \Gamma\backslash\mathbb{B}$ to be the occult period map. By [ACT02, ACT11], [LS07], [Kon00, Kon02]:

Theorem 2.1 (Allcock-Carlson-Toledo, Looijenga-Swierstra, Kondō)**.** *The occult period map $\mathscr{P}\colon \mathcal{M} \longrightarrow \Gamma\backslash\mathbb{B}$ is an open embedding between quasi-projective varieties.*

Noticing that both two sides of $\mathscr{P}$ have natural orbifold structures, Kudla and Rapoport conjectured:

Conjecture 2.2 (Kudla-Rapoport)**.** *The occult period map $\mathscr{P}$ identifies the natural orbifold structures of $\mathcal{M}$ and $\mathscr{P}(\mathcal{M}) \subset \Gamma\backslash\mathbb{B}$.*

I confirmed those conjectures in [Zhe17]. For the two cases of cubics, the key step is to show:

Theorem 2.3. *For X a smooth cubic fourfold, the automorphism group of polarized Hodge structure on $H^4(X,\mathbb{Z})$ is naturally identified with $\mathrm{Aut}(X)$. For T a smooth cubic threefold, the automorphism group of the polarized intermediate Jacobian $J(T)$ is naturally identified with $\mathrm{Aut}(T) \times \mu_2$.*

The first part of theorem 2.3 and the global Torelli theorem for cubic fourfolds imply that, for any two smooth cubic fourfolds X_1, X_2 with $\phi\colon H^4(X_1,\mathbb{Z}) \cong H^4(X_2,\mathbb{Z})$ preserving the polarized Hodge structures on both sides, there exists a linear isomorphism $f\colon X_2 \cong X_1$ such that $f^* = \phi$. This strong global Torelli was claimed in [Cha12] using the Beauville-Donagi construction and Verbitsky's Torelli theorem for hyper-Kähler manifolds (see [Mar11]). Our approach, which made use of Luna's étale slice theorem, was more direct.

2.2. **Future plan.** A long-term question is whether theorem 2.3 holds in general. Precisely:

Question 2.4. *Let be given a smooth degree d n-fold X with $d \geq 3$ and $n \geq 2$. If n is even, we have a group morphism $\varphi\colon \mathrm{Aut}(X) \longrightarrow \mathrm{Aut}_{hs}(H^n(X,\mathbb{Z}), H^{\frac{n}{2}})$, where Aut_{hs} stands for group of Hodge isometries and H is the hyperplane class. If n is odd, we have $\varphi\colon \mathrm{Aut}(X) \longrightarrow \mathrm{Aut}_{hs}(H^n(X,\mathbb{Z}))/\{\pm 1\}$. For which (d,n) is the morphism φ surjective?*

It is worthwhile to mention that for all but finitely many (d,n), the morphisms φ are known to be injective. See [JL17] (proposition 2.12).

The graph of an automorphism of X is a cycle on $X \times X$ of dimension n. By Künneth theorem, a Hodge isomotry of $H^n(X,\mathbb{Z})$ gives rise to a Hodge class in $H^{2n}(X \times X,\mathbb{Z})$. Question 2.4 asks when certain Hodge class on $X \times X$ is induced by the graph of an automorphism of X.

3. Automorphism groups of $K3$ surfaces and cubic fourfolds

A consequence of the Torelli theorem for $K3$ surfaces is that there is a close connection between the automorphism group $\mathrm{Aut}(S)$ of a $K3$ surface S and the Hodge isometries of $H^2(S,\mathbb{Z})$. Nikulin [Nik79a] started a systematic study of the possible finite automorphism groups for $K3$ surfaces by means of lattice theory ([Nik79b]). This topic culminated with the celebrated result of Mukai [Muk88] relating the classification of the finite groups of symplectic automorphisms acting on $K3$ surfaces with certain subgroups of the Mathieu group M_{23}. We denote $\mathrm{Aut}_s(S)$ to be the symplectic automorphism group of a $K3$ surface S.

Theorem 3.1 (Mukai)**.** *There exists a bijection between the finite groups $G \subset Aut_s(S)$ acting symplectically on some projective $K3$ surface S and the subgroups G of the Mathieu group M_{23} with the induced action on $\{1, 2, \ldots, 24\}$ having at least 5 orbits.*

Mukai's work started a fascinating new topic, the Mathieu Moonshine conjecture, which relates the elliptic genus of a $K3$ surface and irreducible representations of the Mathieu group M_{24}. This will be discussed in section 3.2.1.

Kondō [Kon98] has simplified Mukai's proof by relating the classification problem to (the automorphisms of) the Niemeier lattices. Kondō's approach avoids the Leech lattice, but it turns out that a related construction that involves only the Leech lattice behaves more uniformly and adapts to higher dimensions. See [Huy16].

3.1. **Symplectic automorphism groups of smooth cubic fourfolds.** At Radu Laza's suggestion, we started working on classification of automorphism groups of smooth cubic fourfolds and obtained the following:

Theorem 3.2 (Laza-Zheng)**.** *Let X be a smooth cubic fourfold with symplectic automorphism group G (symplectic means that G acts trivially on $H^{3,1}(X)$). Let $\mathcal{F}$ be the moduli space of cubic fourfold with the specified symplectic action by group G. Then we have and only have the following situations:*

(1) Case $\dim(\mathcal{F}) = 20$, $G = 1$.

(2) Case $\dim(\mathcal{F}) = 12$, $G = \mathbb{Z}/2$.

(3) Case $\dim(\mathcal{F}) = 8$, $G = (\mathbb{Z}/2)^2$ *or* $G = \mathbb{Z}/3$.

(4) Case $\dim(\mathcal{F}) = 6$, $G = S_3$ *or* $\mathbb{Z}/4$.

(5) Case $\dim(\mathcal{F}) = 5$, $G = D_8$.

(6) Case $\dim(\mathcal{F}) = 4$, $G = A_{3,3}, D_{12}, A_4, D_{10}$.

(7) Case $\dim(\mathcal{F}) = 3$, $G = S_4$ *or* Q_8.

(8) Case $\dim(\mathcal{F}) = 2$, $G = 3^{1+4} : 2, A_{4,3}, A_5, 3^2.4, S_{3,3}, F_{21}, Hol(4)$ *or* QD_{16}.

(9) Case $\dim(\mathcal{F}) = 1$, $G = 3^{1+4} : 2.2, A_6, \mathrm{PSL}(2, \mathbb{F}_7), S_5, M_9, 3^2.D_8$ *or* T_{48}.

(10) Case $\dim(\mathcal{F}) = 0$, $G = 3^4 : A_6, A_7, 3^{1+4} : 2.2^2, M_{10}, \mathrm{PSL}(2, \mathbb{F}_{11})$ *or* $(3 \times A_5) : 2$.

Previously, the best result was a classification of prime-power-order automorphisms by Fu ([Fu16]). Actually, we did more in [LZ18]. We found explicit equations for cubic fourfolds in many cases. Furthermore, We considered the uniqueness problem for a given G, namely, whether the moduli of cubic fourfolds with action of G is irreducible. In particular, we gave precise classification of the cubic fourfolds when $\dim(\mathcal{F}) = 0$.

An outline of the proof of theorem 3.2 is as follows. Let be given a smooth cubic fourfold X with symplectic automorphism group $G = \mathrm{Aut}_s(X)$. The covariant lattice $S_G(X)$ of the induced action of G on $H^4(X, \mathbb{Z})$ is a primitive sublattice of the Leech lattice $\mathbb{L}$. Via this, the group G is a subgroup of the Conway group $\mathrm{Co}_0 = \mathrm{Aut}(\mathbb{L})$. There is a classification [HM16] of subgroups of Co_0 which is the maximal one fixing certain sublattice of $\mathbb{L}$. Then the list in [HM16] contains all symplectic automorphism groups of smooth cubic fourfolds. We were able to give precise criterion on when a subgroup of Co_0 comes from a symplectic action on smooth cubic fourfold. Finally, we applied Nikulin's criterion on existence of even lattices with given genus ([Nik79b]) to determine all possibilities of G.

As shown in theorem 3.2, there are 34 possibilites of finite groups realized as symplectic automorphism group of smooth cubic fourfolds ([LZ18]). This offers examples for which we can apply the result in [YZ18b] to obtain identifications between GIT-compactifications and Looijenga compactifications of type IV domains. See section 4 for more on this part of my work with Yu [YZ18b].

3.2. **Future work.**

3.2.1. *Automorphism groups and cubics.* There are automorphisms of smooth cubic fourfolds which are not symplectic. Therefore, after the classification of symplectic automorphism groups, it is natural to ask:

Question 3.3. *Which groups appear as automorphism groups of smooth cubic fourfolds?*

In fact, we have already had new discoveries regarding to those groups based on the results of [LZ18]. We proposed a procedure to figure out all automorphism groups of smooth cubic fourfolds, which was accessible by programming. However, we are expecting for more conceptual structures to reduce the calculation.

The classification of automorphism groups of cubic threefolds is far from complete. Notice that a smooth cubic threefold gives rise to a smooth cubic fourfold with a specified order three automorphism. We ask:

Question 3.4. *With the help of classification results on automorphism groups of cubic fourfolds, can we obtain new results on classification of automorphism groups of cubic threefolds?*

3.2.2. *Automorphism groups and hyper-Kähler manifolds.* Finite groups of symplectic automorphism of hyper-Kähler manifolds of type $K3^{[2]}$ are classified in [HM14]. There are 15 such groups which are maximal. An appealing question raised by the authors of [HM14] is:

Question 3.5 (Höhn-Mason)**.** *What are the explicit descriptions of the hyper-Kähler manifolds with maximal finite symplectic automorphism groups?*

Considering symplectic automorphism groups of polarized hyper-Kähler manifolds which are Fano scheme of lines on smooth cubic fourfolds ([BD85]), we obtain 6 groups out of the 15. Besides the Beauville-Donagi construction ([BD85]), we have other constructions ([DV10], [O'G06], [RV17]) of 20-dimensional families of polarized hyper-Kähler manifolds of type $K3^{[2]}$.

Question 3.6. *Can we use those constructions to obtain type $K3^{[2]}$ hyper-Kähler with maximal symplectic automorphism groups?*

3.2.3. *Mathieu moonshine.* Eguchi, Ooguri and Tachikawa ([EOT11]) observed that the elliptic genus of a $K3$ surface has a decomposition with coefficients in terms of the dimensions of irreducible representations of the Mathieu group M_{24}. This is formulated as the Mathieu Moonshine conjecture. EOT's observation leads to a more general conjecture, called the Umbral Moonshine conjecture, which is rigorously proved in [DGO15]. However, the intrinsic nature of those phenomena is still a mystery.

More Moonshine phenomena were found for some simple subgroups of M_{24}, for instance, there are [EH12] about $\mathrm{PSL}(2, \mathbb{F}_{11})$, and [EH13] about the mathieu group M_{12}. We observed that $\mathrm{PSL}(2, \mathbb{F}_{11})$ appears as symplectic automorphism group of certain cubic fourfold (namely, the triple cover of $\mathbb{P}^4$ branched along the Klein cubic threefold).

Question 3.7. *Can we establish relations between automorphism groups of $K3$ surfaces/cubic fourfolds/hyper-Kähler manifolds and Moonshine phenomena for finite subgroups of M_{24} or Co_0?*

4. Moduli theory related to $K3$ surfaces and cubic fourfolds

4.1. **Moduli of symmetric cubic fourfolds.** The occult period map realizes the moduli of s-mooth cubic threefolds as an arrangement complement in an arithmetic ball quotient of dimension 10. An extension of the occult period map identifies the GIT-compactification and the Looijenga compactification. These were proved in [ACT11] and [LS07] independently. The occult period map essentially views cubic threefold as a cubic fourfold with a specified order three automorphism. In [YZ18b], we generalize the work in [ACT11] to cubic fourfolds with any specified group action.

Theorem 4.1 (Yu-Zheng)**.** *Let be given a smooth cubic fourfold X with an action of a finite group G. We can use method in* GIT *to construct a moduli space $\mathcal{F}$ of smooth cubic fourfolds with the same type of action by G. This space $\mathcal{F}$ is an irreducible normal quasi-projective variety. We have a Hermitian symmetric domain $\mathbb{D}$ (which is either a ball or type* IV *domain) with a proper discontinuously action by an arithmetic group Γ, such that there exists a period map*

$$\mathscr{P}\colon \mathcal{F} \longrightarrow \Gamma\backslash\mathbb{D}$$

which is an algebraic open embedding. Moreover, there is an extension:

$$\mathscr{P}\colon \overline{\mathcal{F}} \cong \overline{\Gamma\backslash\mathbb{D}}^{\mathcal{H}_*}$$

where $\overline{F}$ is the GIT*-compactification of $\mathcal{F}$, $\mathcal{H}_*$ is a Γ-invariant hyperplane arrangement in $\mathbb{D}$, and $\overline{\Gamma\backslash\mathbb{D}}^{\mathcal{H}_*}$ is the Looijenga compactification of $\Gamma\backslash(\mathbb{D}-\mathcal{H}_*)$.*

There are two aspects I would like to emphasize. Firstly, in [YZ18b] we were able to deal with any finite group actions, which provided a bunch of new examples of locally symmetric varieties with modular meaning. Secondly, to show the identifications between GIT-compactifications and Looijenga compactifications, we adopted a new approach instead of the standard one, and our approach turned out to be more natural and universal in general situation. The standard approach needs a calculation of self-intersections of the members in the hyperplane arrangement $\mathcal{H}_*$. Our approach avoided such calculations, and used instead certain functorial properties of GIT-compactifications, Baily-Borel compactifications and Looijenga compactifications, together with the characterization of the image of the period map for smooth cubic fourfolds (by Looijenga [Loo09] and Laza [Laz10] independently).

4.2. **Moduli of singular sextic curves.** We extended our ideas and techniques in [YZ18b] to characterize moduli spaces of nodal sextic curves. See [YZ18a].

The key idea is that the double cover of $\mathbb{P}^2$ ramifying along a nodal sextic curve is a nodal $K3$ surface. The resolution of a nodal $K3$ surface is a smooth $K3$ surface with a natural lattice polarization. There is a natural stratification on the moduli space of plane sextic curves induced by the number of nodes. Let T be a singular type, which corresponds to an irreducible component of certain strata. Let $\mathcal{F}_T$ be the moduli space of sextic curves of type T and $\overline{\mathcal{F}}_T$ the GIT-compactification, which are constructed via geometric invariant theory. The period map of the corresponding lattice-poarized K3 surfaces gives a morphism $\mathscr{P}\colon \mathcal{F}_T \longrightarrow \Gamma\backslash\mathbb{D}$. Here $\Gamma\backslash\mathbb{D}$ is an arithemetic quotient of type IV domain. There is a Γ-invariant hyperplane arrangement $\mathcal{H}_*$ on $\mathbb{D}$.

Theorem 4.2 (Yu-Zheng)**.** *For any singular type T, the period map $\mathscr{P}\colon \mathcal{F}_T \longrightarrow \Gamma\backslash\mathbb{D}$ is an algebraic open embedding and extends to an isomorphism between projective varieties $\mathscr{P}\colon \overline{\mathcal{F}}_T \cong \overline{\Gamma\backslash\mathbb{D}}^{\mathcal{H}_*}$.*

For most choices of T, the hyperplane arrangements $\mathcal{H}_*$ are empty and hence we identify $\overline{\mathcal{F}}_T$ with the Baily-Borel compactifications $\overline{\Gamma\backslash\mathbb{D}}^{bb}$ of $\Gamma\backslash\mathbb{D}$. In [YZ18a], we gave a criterion on when $\mathcal{H}_*$ is empty.

As special cases, we recover works concerning moduli of six lines ([MSY92]), moduli of pairs consisting of a quintic curve and a line ([Laz09]), and moduli of triples consisting of a quartic curve and two lines ([GMGZ17]). Our work has close relation with low genus curves and del Pezzo surfaces. For example, the normalization of a sextic curve with 4 nodes has genus 6. And a generic genus 6 curve lies on a del Pezzo surface of degree 5. See [AK11].

4.3. **Future plan.**

4.3.1. *Structure of the modular varieties.* From [YZ18b, YZ18a], we obtain a bunch of locally symmetric varieties with modular meaning.

Question 4.3. *Can we say more about structures of (the Baily-Borel or Looijenga compactifications of) those modular varieties, for instances, their cohomology groups, Picard groups, Chow rings, homotopy groups, rationalities, Hasse-Weil zeta functions and so on?*

4.3.2. *Teichmüller curve.* Another potential direction involves of dynamical system. A Teichmüller curve is a quasi-projective complex geodesic in $\mathcal{M}_g$ (the moduli of Riemann surfaces of genus g) with Teichmüller metric. In [McM03], McMullen constructed an infinite series of Teichmüller curves in $\mathcal{M}_2$. An explicit description of those Teichmüller curves are given via theory of translation surface, which is now an active area in dynamical system. The Teichmüller curves constructed in [McM03] lie on Hilbert modular surfaces. From [YZ18b, YZ18a], we can produce modular curves and modular surfaces as moduli spaces.

Question 4.4. *Can we produce Teichmüller curves via moduli of $K3$ surfaces or cubic fourfolds with additional structures? If so, how are the $K3$ surfaces/cubic fourfolds connected to translation surfaces? Can we obtain interesting connection to number theory?*

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